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Macroscopic quantum information channel via the polarization-sensitive interaction between the light and spin subsystems

O.S. Mishina, D.V. Kupriyanov,

*Department of Theoretical Physics, State Polytechnic University, 195251, St.-Petersburg, Russia**

E.S.Polzik

QUANTOP - Danish Quantum Optics Center, Niels Bohr Institute, 2100 Copenhagen, Denmark†

Abstract

We discuss the quantum information channel, which is based on coherent and polarization sensitive interaction of light and atomic spin waves. We show that the joint Heisenberg dynamics of the polarization Stokes components of light and of the angular momenta of atoms has a wave nature and can be properly described in terms of the macroscopic polariton-type spin wave created in the sample. The principles of the quantum memory and readout protocols via the wave coupling of the output time modes in the light subsystem and the output spatial modes in the spin subsystem are demonstrated.

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*Electronic address: ksana@quark.stu.neva.ru

†Electronic address: polzik@phys.au.dk

I. INTRODUCTION

Quantum information processing based on continuous variables as an alternative to the discrete quantum schemes has been intensely studied during the last decade, see, for example, the review [1] and reference therein. Spin oriented atomic ensemble and polarized coherent light can be considered as convenient physical objects for storage and for transport of the quantum information, which can be written in the fluctuations of their polarizations. There are two experimental demonstrations of the entanglement and memory protocols in Refs.[2, 3], where the dynamical coupling of the spin fluctuations of an atomic ensemble consisting of cesium atoms with the polarization fluctuations of linearly polarized light via Faraday effect were implemented. The main feature and certain advantage of those protocols are in that only collective integral variables of light and of atomic subsystems contributed in the quantum fluctuation interchange. However, as was recently reported in Ref.[4], in a more general situation the light and spin subsystems develop the polariton-type spin wave dynamics in the sample. That means that the dynamics of the Stokes polarization components and of the atomic spins becomes coupled not integrally and collectively but locally in time as well as in space during the whole interaction cycle. In this case the quantum correlations are spread among all the possible time and spatial polariton modes created in the sample. In this report we further develop the ideas of Ref.[4] and show how they could be applied for organizing the readout and memory protocols based on the wave dynamics of the process.

II. MACROSCOPIC POLARITON-TYPE SPIN WAVE

Consider an off-resonant pulse of radiation probing the spin-polarized atomic ensemble during the short time interval so that incoherent scattering is negligible. Then, as shown in [4], the relevant Heisenberg operators describing the light and atomic subsystems, and

responsible for the quantum correlations dynamics, obey the following wave-type equations

$$\begin{aligned}
\left[\frac{\partial}{\partial z}+\frac{1}{c}\frac{\partial}{\partial t}\right]\hat{\Xi}_1(z,t) &= -\kappa_2\,\hat{\Xi}_2(z,t) \;+\; 2\beta\,\bar{\Xi}_3\,\hat{\mathcal{J}}_z(z,t)\\
\left[\frac{\partial}{\partial z}+\frac{1}{c}\frac{\partial}{\partial t}\right]\hat{\Xi}_2(z,t) &= \quad \kappa_2\,\hat{\Xi}_1(z,t) \;-\; 2\epsilon\,\bar{\Xi}_3\,\hat{\mathcal{J}}_y(z,t)\\
\frac{\partial}{\partial t}\hat{\mathcal{J}}_z(z,t) &= \quad \Omega\,\hat{\mathcal{J}}_y(z,t) \;-\; \epsilon\bar{\mathcal{J}}_x\,\hat{\Xi}_1(z,t)\\
\frac{\partial}{\partial t}\hat{\mathcal{J}}_y(z,t) &= -\Omega\,\hat{\mathcal{J}}_z(z,t) \;+\; \beta\bar{\mathcal{J}}_x\,\hat{\Xi}_2(z,t) \qquad (1)
\end{aligned}$$

In these equations the light pulse with 100% linear polarization along x-direction propagates through the sample along z-direction. The atoms also have 100% orientation of their angular momenta along the x-direction. The field subsystem is described by the Heisenberg operators for two polarization Stokes components $\hat{\Xi}_1(z,t)$ and $\hat{\Xi}_2(z,t)$. The first Stokes component $\hat{\Xi}_1(z,t)$ is responsible for imbalance between photon fluxes of the modes linearly polarized along ξ and η axes rotated with respect to x and y directions by $\pi/4$ angle, and the second component $\hat{\Xi}_2(z,t)$ is responsible for imbalance in the right-hand and the left-hand polarizations. The third Stokes component $\bar{\Xi}_3$ responsible for imbalance in x- and y-type polarizations is a quantum integral of motion and considered as external (non-operator) parameter in equations (1). The atomic subsystem is described by the operators $\hat{\mathcal{J}}_z(z,t)$ and $\hat{\mathcal{J}}_y(z,t)$, which are the Heisenberg operators for the spatial distribution of the transverse fluctuations of the angular momenta in respect to z and y axes. The spin projection $\bar{\mathcal{J}}_x$ on x direction is another quantum integral of motion and is also considered a c-number parameter.

The system of equations (1) describes the coupled wave-type dynamics of the Stokes components of the light subsystem and of the transverse fluctuations of the atomic angular momenta. Since the field subsystem consists of many photons and the spin subsystem consists of macroscopic number of individual atomic spins, we shall call such a wave as a macroscopic polariton-type spin wave. This polariton wave describes the mesoscopically smooth dynamics for the interacting microscopic quantum variables of the light and spin subsystems. Such a wave represents a manifestly quantum object, which exists only at the level of quantum description of the field and spin fluctuations. While deriving equations (1) it was assumed that atoms preserve their location during the interaction cycle and there is no destruction of the coherent wave dynamics coming from the random atomic motion. This important condition can be fulfilled, for example, for interaction duration longer than tenth

of milliseconds for an ensemble consisting of ultracold atoms. Alternatively for hot atoms in a gas cell the interaction time should be shorter than a few microseconds.

The following two important parameters govern the dynamics of the process. The first one is β, which is the angle of the polarization rotation of the probe light due to Faraday effect per one spin flip in the ensemble in z-direction. The second is ϵ, which is the ellipticity induced in the propagating light by the atomic sample due to Cotton-Mouton effect per one spin flip in y-direction . Frequency $\Omega = \Omega_0 + \Omega_2$ combines the regular precession caused by the external magnetic field Ω_0 with the frequency of light-induced shift of the Zeeman sublevels Ω_2. Parameter κ_2 is responsible for birefringence effects with respect to x- and y-type polarizations of the probe, i.e. for the unitary transformation of linear polarization to circular polarization and vice versa.

In general the system of equations (1) can be solved only numerically. But in a special case it can be solved analytically if at least one of the parameters, either Ω or κ_2 approaches zero. In practice this can be happened if, for example, the magnetic field compensates the contribution of the light shift so that $\Omega_0 = -\Omega_2$. The solution of the system (1) for this special case will be discuss in details elsewhere. Here we restrict our further discussion to an even simpler model where $\kappa_2 = 0$ and $\Omega = 0$ simultaneously. Although it is not trivial to compensate the difference in the refractive indices for x- or y-type polarized probe propagating in the spin polarized atomic ensemble, it is very appealing to consider this particular case, due to the following two reasons. 1) There are no quantum correlations under the above conditions induced between the field and atomic subsystems due to regular spin precession and due to average birefringence of the sample. 2) Such an approximation lets us clear identify the basic features of the macroscopic spin polariton dynamics and their potential importance for the respective quantum information channel. In addition, as a technical simplification, we will assume that the propagation time through the sample is negligibly small, and the retardation effects are unimportant in the frequency domain we are going to discuss.

III. ENTANGLEMENT BETWEEN THE LAPLACE MODES

In experiment the light pulse probes an atomic sample of the length L during the finite time T. However, because the wave dynamics is developing only in the forward direction

in space as well as in time, we can formally extend this process up to infinite interaction time and consider the probe light propagating in a semi-infinite medium. Then parameter L can be associated with a selected layer in such a medium located at the coordinate $z = L$ and after interaction time T the state of the system is considered at the moment $t = T$. Such an extension lets us define the following Laplace modes for the field and for the atomic subsystem

$$\begin{aligned} \hat{\Xi}_i^{\rm out}(s) \;&=\; \hat{\Xi}_i(L,s) = \int_0^\infty dt\, e^{-st}\, \hat{\Xi}_i(L,t) \\ \hat{\mathcal{J}}_\mu^{\rm out}(p) \;&=\; \hat{\mathcal{J}}_\mu(p,T) = \int_0^\infty dz\, e^{-pz}\, \hat{\mathcal{J}}_\mu(z,T) \end{aligned} \tag{2}$$

for $i = 1, 2$ and $\mu = z, y$ respectively. In the above transformations we consider the outgoing field operators at point $z = L$ at any time and define the Laplace s-mode for them. The outgoing atomic operators are considered at a selected moment of time $t = T$ but for an arbitrary spatial location and are described in terms of the spatial Laplace p-mode.

The solution of the system (1) can be conveniently rewritten using relations between the Laplace modes in the following form

$$\begin{aligned} e^{-p(s)L}\, \hat{\Xi}_1^{\rm out}(s) \;-\; \frac{2\beta\bar{\Xi}_3}{s}\, e^{-sT}\, \hat{\mathcal{J}}_z^{\rm out}(p(s)) \;=& \\ \int_T^\infty dt\, e^{-st}\, \hat{\Xi}_1^{\rm in}(t) \;-\; \frac{2\beta\bar{\Xi}_3}{s} \int_L^\infty dz\, e^{-p(s)z}\, \hat{\mathcal{J}}_z^{\rm in}(z) & \\ e^{-p(s)L}\, \hat{\Xi}_2^{\rm out}(s) \;+\; \frac{2\epsilon\bar{\Xi}_3}{s}\, e^{-sT}\, \hat{\mathcal{J}}_y^{\rm out}(p(s)) \;=& \\ \int_T^\infty dt\, e^{-st}\, \hat{\Xi}_2^{\rm in}(t) \;+\; \frac{2\epsilon\bar{\Xi}_3}{s} \int_L^\infty dz\, e^{-p(s)z}\, \hat{\mathcal{J}}_y^{\rm in}(z) & \end{aligned} \tag{3}$$

The initial Heisenerg operators $\hat{\Xi}_i^{\rm in}(t)$ and $\hat{\mathcal{J}}_\mu^{\rm in}(z)$ of the field and atomic subsystems contribute on the right hand side and are defined in their original form as functions on time and spatial coordinates. There are two additional relations, not shown here, which together with the relations (3) lead to the full solution in the Laplace form. The main feature of the solution (3) is that s- and p- Laplace modes are not independent one on another but coupled by the dispersion relation caused by the wave nature of the process

$$\begin{aligned} p \;&=\; p(s) \;=\; \frac{A}{s}, \\ A \;&=\; -2\beta\,\epsilon\,\bar{\Xi}_3\,\bar{\mathcal{J}}_x \end{aligned} \tag{4}$$

And because of complete symmetry of the problem similar dispersion relation can be written for $s = s(p)$.

As clearly seen from the solution (3) in an extended medium $L \to \infty$ and after long interaction time $T \to \infty$ there will be entanglement developed between the temporal and spatial dynamics inside the polariton wave mode. When the field and atomic subsystems are separated the mode entanglement would manifest itself in either entanglement or swapping of the outgoing quantum states. The wave nature of such type of quantum correlations will be visualized after a certain spectral selection in the light subsystem and a spatial spectral selection in the spin subsystem are made.

The important parameter which determines the type of the output quantum correlations is the polariton group velocity. To define this velocity the inverse Laplace transformation should be written in terms of a Fourier integral, with the following parametrization of the Laplace modes: $s = -i\omega$ and $p = iq$. Then the transport dynamics of the correlation wave is characterized by the group velocity

$$v_g \;=\; \frac{d\omega}{dq} \;=\; -\frac{A}{q^2} \tag{5}$$

As one can see the group velocity can be either positive or negative. The latter case appears when the product $\beta\epsilon < 0$, as, in example of alkali atoms, can occur for the probe in the blue wing of D_2-line. For such a specific situation there is no stationary point in the corresponding wavepacket expansion. The quantum fluctuation are exponentially enhanced in space as well as in time with preserving their quantum correlations. This case requires a special discussion. Below we consider the alternative and a more typical for the standard wave dynamics situation when the group velocity is positive and $\beta\epsilon > 0$. In case of alkali atoms this relates to the red wing of D_2-line.

IV. QUANTUM MEMORY AND READOUT PROTOCOLS

Consider an experimental situation when a low-frequency mode with the frequency ω, such that $\omega T < 1$, is detected in the output polarization state of the transmitted light. For this mode, according to the dispersion law (4) and (5), the group velocity can be quite low. As consequence the input quantum state of the entire system will be transport through the sample as a slow propagating wavepacket. Then the output Heisenberg operators of the light subsystem will be actually formed in the readout process of the input quantum operators of atomic spins and preferably of those spatial components which are distributed

near respective high-frequency spatial mode $q = A/\omega$.

This can be demonstrated by the graphs plotted in figure 1. All the dependencies are the expectation variances for the following target integral observables

$$\hat{\Xi}_i^{\text{out}} = \int_0^T dt\, \cos(\omega t)\, \hat{\Xi}_i^{\text{out}}(t) \tag{6}$$

for $i = 1, 2$. The calculations are performed for the red wing of D_2-line of ^{133}Cs, for detuning of the probe about -1200 MHz from the $F_0 = 4 \rightarrow F = 5$ hyperfine transition. The composition parameter $-ALT$ is varied from 0 (left border of the graph) to the level of 2 (right border of the graph) so that the chosen in our calculations detecting frequency $\omega T = 0.5$ would be asymptotically coupled with the spatial mode with the wave number $qL = 4$, in the way explained in the previous section. The input quantum states of the light and spin subsystems obey Poissonian statistics. The blue curve shows how the input quantum fluctuations of the Stokes components are mapped onto the output state. As one can see for the extended medium with high optical activity βJ (where $J = \bar{\mathcal{J}}_x L$ is the total angular momentum of the sample) the contribution of the input field fluctuations becomes negligible. The main impact on the output variances is made by the input fluctuations of the spin subsystem. It also follows from the graphs that the variance of $\hat{\Xi}_1^{\text{out}}$ component is much bigger than $\hat{\Xi}_2^{\text{out}}$ and the latter is below the standard quantum deviation. This is a direct consequence of inequality $\beta \gg \epsilon$, which is typical in the case of D_2 transition of alkali atoms. So the mapping of the state of light is in this case combined with the squeezing operation on the state. The output polarization state of the probe light in the low-frequency domain of its fluctuation spectrum becomes squeezed and the high-frequency spatial modes of the spin states $\hat{J}_z^{\text{in}}$ and $\hat{J}_y^{\text{in}}$ are respectively mapped into the integral modes of the $\hat{\Xi}_1^{\text{out}}$ and $\hat{\Xi}_2^{\text{out}}$ Stokes components.

Let us reverse the experimental situation and let the quantum state of the spin subsystem be controlled by the high-frequency polarization fluctuations of light, such that $\omega T > 1$. Then the quantum memory protocol can be achieved and the quantum state of these modes can be mapped into the output low-frequency spatial modes of the spin subsystem. Indeed, in this case the transfer of input correlations is fast because the polariton group velocity v_g is high enough. The original input quantum state of the system will be transported out of the sample with the polariton wave. Then the low-frequency spatial modes (with $qL < 1$) of the output spin state are mainly formed via accumulation of the input quantum fluctuations

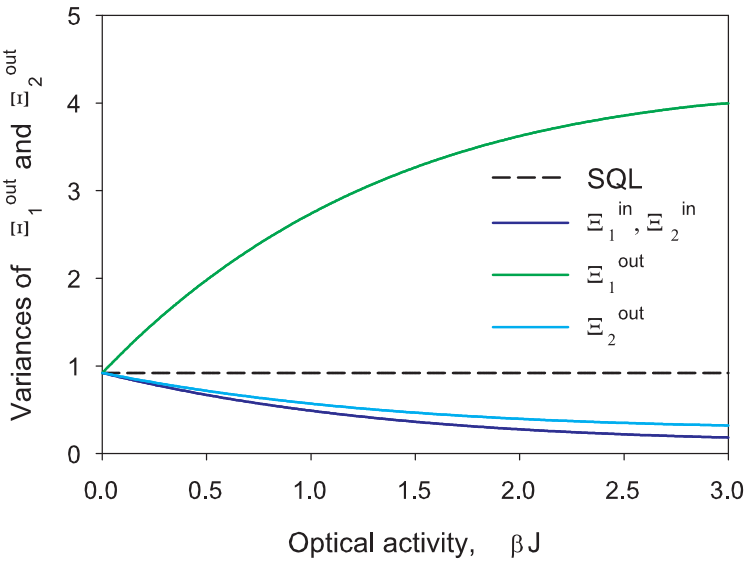

FIG. 1: The variances of the integral output Stokes components, defined by Eq.(6) for $\omega T = 0.5$, as a function of the optical activity of the sample βJ. The dotted curve indicates the vacuum noise (standard) quantum limit (SQL). The blue curve indicates the contribution of the input Stokes components.

of the transmitted light. Because of complete symmetry of the equations (1) with respect to the light and spin subsystems, the memory protocol can be visualized by the same graphs shown in figure 1 with the following change in notation. The target observables should be now associated with the spin subsystem

$$\hat{J}_{\mu}^{\rm out} = \int_0^L dz\, \cos(qz)\, \hat{\mathcal{J}}_{\mu}^{\rm out}(z) \tag{7}$$

for $\mu = z, y$. The blue curve indicates the reduction of the input state mapping onto these observables. The abscissa of the plot should be associated with $\beta\bar{\Xi}_3 T$, which is the angle of the collective spin rotation if the probe light were circular polarized. After change in notation the graphs of figure 1 show how the initial polarization quantum state of light, which existed in the high-frequency domain of its fluctuation spectrum, can be mapped into the integral output quantum state of atomic spins. Asymptotically (for the parameters used in our calculations) this results in the mode coupling between $\omega T = 4$ time mode of the Stokes variables and the $qL = 0.5$ spatial mode of atomic spins.

V. CONCLUSION

In this report we have discussed how interchange by the polarization quantum states of the light and atomic spins could be implemented via the polarization sensitive interaction between these subsystems. The main result is the identification of the spectral domains, where the quantum states of the input temporal or spatial fluctuations could be transferred from one subsystem to another. If the polarization fluctuations of the light and spin subsystems carried the quantum information it would make possible to create the quantum information protocols for writing in or reading out this information. This specific quantum channel does not exactly copy an original quantum state from one physical carrier to another, but creates a new quantum state in the target system related in a known way to the quantum state originally existing in the source system. This can be done because of multimode nature of interaction in the entire atoms-field system and the protocols are related only to the specially selected spectral domains. In a general case the input-output transformations in the usual space-time representation are expressed via the fundamental solution of the wave-type Heisenberg equations (1) in integral form, see [4]. For interaction limited in space as well as in time the idealized mode description can only approximately approach the real interaction process. Because of the multi-mode character of the input-output transformation the fidelity, usually applied to describe the quality of a quantum information protocol for relatively simple systems, has to be revised in order to become applicable. This work is in progress and will be published elsewhere.

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[1] Samuel L. Braunstein and Peter van Loock, Quantum information with continuous variables, *Rev. Mod. Phys.* **77** (2005), 513.

[2] B. Julsgaard, A. Kozhekin, E.S. Polzik, Experimental long-lived entanglement of two macroscopic objects, *Nature* **413** (2001), 400.

[3] B. Julsgaard, J. Sherson, J. Fiurasek, J.I. Cirac, and E.S. Polzik, Experimental demonstration of quantum memory for light, *Nature* **432** (2004), 482.

[4] D.V. Kupriyanov, O.S. Mishina, I.M. Sokolov, B. Julsgaard, E.S. Polzik, Multimode entanglement of light and atomic ensembles via off-resonant coherent forward scattering, *Phys. Rev. A* **77** (2005), 032348.